Thermal-Stress-Free Fasteners for Joining Orthotropic Materials

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Hot structures fabricated from orthotropic materials are an attractive design option for future high-speed vehicles. Joining subassemblies of these materials with standard cylindrical fasteners can lead to loose joints or highly stressed joints due to thermal stress. A method has been developed to eliminate thermal stresses and maintain a tight joint by shaping the fastener and mating hole. This method allows both materials (fastener and structure), with different coefficients of thermal expansion (CTE's) in each of the three principal material directions, to expand freely with temperature yet remain in contact. For the assumptions made in the analysis, the joint will remain snug, yet free of thermal stress at any temperature. Finite-element analysis was used to verify several thermal-stress-free fasteners and to show that conical fasteners, which are thermal-stress-free for isotropic materials, can reduce thermal stresses for transversely isotropic materials compared to a cylindrical fastener. Equations for thermal-stress-free shapes are presented and typical fastener shapes are shown.

Nomenclature

c_1,c_2	= constants in partial differential equation
c_3,c_4	= integration constants for characteristic equations
E	= modulus of elasticity
L	= length
m,n	= exponents in Eq. (18)
p	= parameter that determines characteristic shape in x-z plane
q	= parameter that determines characteristic shape in y-z plane
r,θ,z	= cylindrical coordinates
r_0	= radius of fastener shank
T	= temperature
T_0	= initial temperature
TSF	= thermal-stress-free
x,y,z	= rectangular Cartesian coordinates
z_0	= length of fastener shank or thickness of washer
α	= coefficient of linear thermal expansion (CTE)
ν	= Poisson's ratio
σ	= stress
τ	= shear stress

Subscripts

max	= maximum
pin	= referring to a cylindrical pin
r	= in the r direction
x	= in the x direction
y	= in the y direction
z	= in the z direction
θ	= in the θ direction
1	= material 1
2	= material 2

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Introduction

HOT structures are an attractive design option for some structural components of future high-speed vehicles to reduce weight and increase vehicle efficiency. An example of such a hot structure[1] is shown in Fig. 1. Most of these hot structures will be fabricated by joining structural subassemblies with metal fasteners. Moreover, carbon or ceramic-based high-temperature materials will likely be used in these structural subassemblies. These high-temperature materials have much lower coefficients of linear thermal expansion (CTE's) than metal fastener materials, which makes it difficult to design a structural joint that remains tight over the operational temperature range. The problems encountered using a standard cylindrical fastener in a high-temperature joint are illustrated in Fig. 2, where the fastener material is assumed to have a much greater CTE than the structural material being joined. If the fastener is snug in the radial direction at room temperature, high thermal stresses will result at elevated temperature. If sufficient radial clearance is left around the fastener so that it is just snug at elevated temperature, then it may be too loose at room temperature. Similarly, if the fastener is axially snug at room temperature, it will be loose at elevated temperature, or it must be highly prestressed at room temperature to remain snug at elevated temperature. The radial thermal stress or axial prestress required to maintain an acceptably tight joint over the operational temperature range may cause premature failure of the joint.

There are several approaches to solving this problem. First, a joint consisting of a standard cylindrical fastener joining pieces of the selected structural material should be analyzed to determine if there is a satisfactory combination of radial clearance and axial prestress at room temperature which will provide acceptable radial thermal stresses and be axially snug at elevated temperature. A second approach is to use a fastener material with a CTE matching the CTE of the structural material closely enough to provide a tight joint with acceptable thermal stresses. However, such a material with the ductility desirable for a fastener, is not presently available for use with carbon and ceramic-based high-temperature structural materials.

If neither of these approaches provides a satisfactory joint, the interface between the fastener and structural materials can be shaped to reduce or eliminate the joint thermal stresses while maintaining a snug fit. Figure 3 illustrates such a joint in which the fastener and structural materials are free to expand thermally while remaining in contact to provide a snug, thermal-stress-free joint at all temperatures. Previous studies[2-4] have identified thermal-stress-free shapes for isotropic and transversely isotropic materials. In this paper, an isotropic material is defined as having the same CTE in all directions, a transversely isotropic material is defined as having the same CTE in all directions in the plane of the material but having a different CTE through-the-thickness of the material, and an orthotropic material is defined as having a different CTE in the direction of each of the three principal material axes.

The objective of this study is to develop a solution for a three-dimensional thermal-stress-free interface between two orthotropic materials and to investigate the behavior of these fasteners using finite-element analysis. This paper contains the solution for a three-dimensional, thermal-stress-free interface between two orthotropic materials at elevated temperature. An algebraic equation is derived for the shape of such an interface, and the assumptions of the analysis are discussed. Results of finite-element analysis of thermal stresses in joints with isotropic and transversely isotropic materials are presented.

Mathematical Analysis

Development of Interface Equation

The objective of this analysis is to find an interface between two orthotropic, homogeneous materials, which have different CTE's in all three principal material directions, along which the two materials will remain in contact, without interference or separation, as temperature changes. The equation for the shape of such an interface is developed in the following discussion.

Consider a continuous surface, $z = f(x,y)$, which separates two orthotropic materials, each with different CTE's along all three principal material axes (see Fig. 4). The material axes of the two materials are assumed to be aligned. All motion due to thermal expansion is assumed to be relative to the origin of the coordinate system, which is parallel to the principal material axes. The CTE's of both materials are assumed to be independent of temperature and location within each material. Also, assume that the temperature is uniform throughout the materials and that no frictional forces act along the interface. Consider a point on the interface at an initial temperature T_0 at which adjacent particles in the two materials are denoted by points 1 and 2. After an infinitesimal change in temperature, the two points, which were adjacent, separate yet remain on a common boundary, as shown in Fig. 4. The separation in the x, y, and z directions are $\mathrm{d}x$, $\mathrm{d}y$, and $\mathrm{d}z$, respectively, and are related to the continuous surface, $z = f(x,y)$, by

$$\mathrm{d}z = \frac{\partial z}{\partial x}\,\mathrm{d}x = \frac{\partial z}{\partial y}\,\mathrm{d}y \tag{1}$$

where

$$\mathrm{d}z = \mathrm{d}z_2 - \mathrm{d}z_1, \qquad \mathrm{d}x = \mathrm{d}x_2 - \mathrm{d}x_1, \qquad \mathrm{d}y = \mathrm{d}y_2 - \mathrm{d}y_1 \tag{2}$$

Using these assumptions, the components of the motion due to thermal expansion for each material are given by the following expressions.

For material 1,

$$\mathrm{d}x_1 = x\alpha_{x_1}\,\mathrm{d}T, \qquad \mathrm{d}y_1 = y\alpha_{y_1}\,\mathrm{d}T, \qquad \mathrm{d}z_1 = z\alpha_{z_1}\,\mathrm{d}T \tag{3}$$

For material 2,

$$\mathrm{d}x_2 = x\alpha_{x_2}\,\mathrm{d}T, \qquad \mathrm{d}y_2 = y\alpha_{y_2}\,\mathrm{d}T, \qquad \mathrm{d}z_2 = z\alpha_{z_2}\,\mathrm{d}T \tag{4}$$

Substitution of Eqs. (2–4) into Eq. (1) produces the following differential equation, which governs the shape of the thermal-stress-free interface between the materials.

$$c_1 x\,\frac{\partial z}{\partial x} + c_2 y\,\frac{\partial z}{\partial y} = z \tag{5}$$

where

$$c_1 = \frac{\alpha_{x_2} - \alpha_{x_1}}{\alpha_{z_2} - \alpha_{z_1}} \qquad c_2 = \frac{\alpha_{y_2} - \alpha_{y_1}}{\alpha_{z_2} - \alpha_{z_1}} \tag{6}$$

Equation (5) is a linear, first-order partial differential equation that can be solved as outlined below, using the method described by Hildebrand.[5] Equation (5) implies the relationship

$$\frac{\mathrm{d}x}{c_1 x} = \frac{\mathrm{d}y}{c_2 y} = \frac{\mathrm{d}z}{z} \tag{7}$$

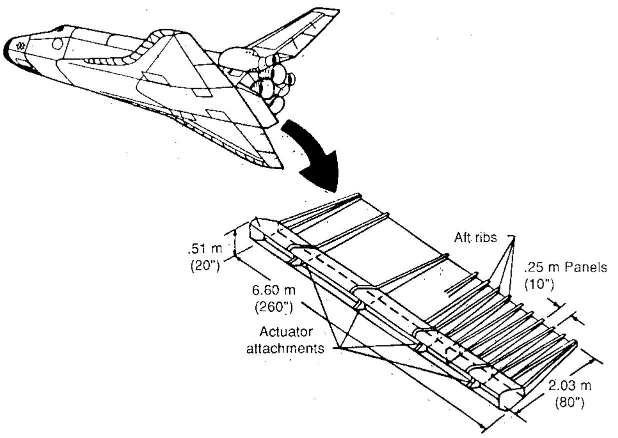
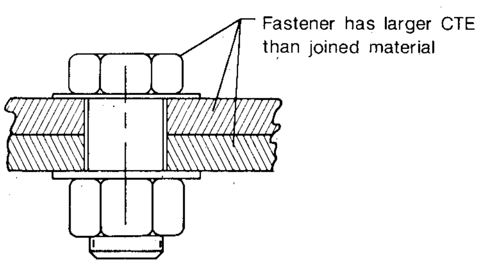


Fig. 1 Conceptual design for carbon-carbon Shuttle Orbiter body flap.



	Joint condition	
	Room temperature	Elevated temperature
Radial direction	Snug Loose	High thermal stresses Snug
Axial direction	Snug Pre-stressed	Loose Snug

Fig. 2 Problem areas for conventional joints in high-temperature applications.

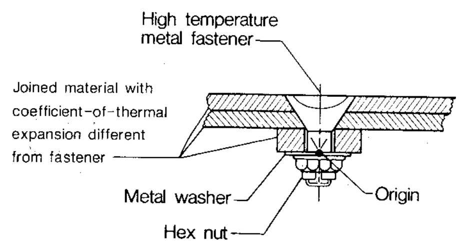


Fig. 3 Thermal-stress-free metal fastener concept for high-temperature joints.

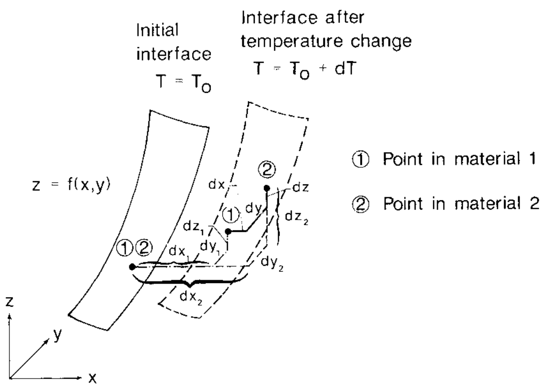


Fig. 4 Thermal expansion along interface between orthotropic materials.

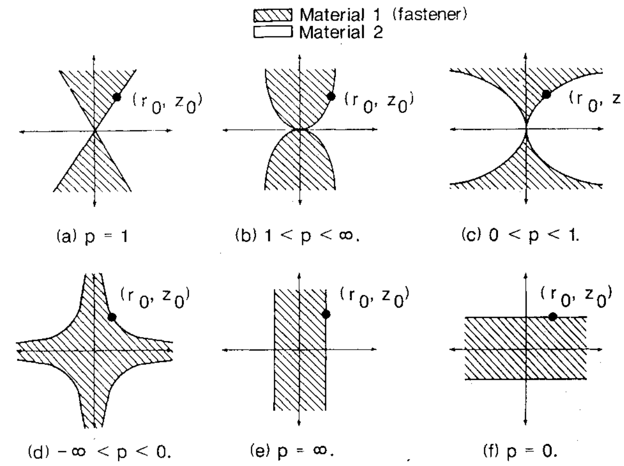


Fig. 5 Thermal-stress-free shapes in x-z plane.

Solution of these two independent ordinary differential equations produces equations defining two surfaces which are given by

$$x^p/y^q = c_3 \qquad x^p/z = c_4 \tag{8}$$

where $p = 1/c_1$, and $q = 1/c_2$ and c_3 and c_4 are constants of integration.

Intersections of the two surfaces defined in Eq. (8) define the characteristic curves of the differential Eq. (5). To define a unique surface, a boundary curve through which these characteristic curves pass must be specified. The following equations define a boundary that represents the intersection of the thermal-stress-free portion of a fastener with a cylindrical shank. At

$$z = z_0, \qquad r_0^2 = x^2 + y^2 \tag{9}$$

where z_0 is the length of the shank or thickness of the washer, and r_0 is the radius of the shank. Substituting $z = z_0$ from Eqs. (9) into Eqs. (8) and solving for x and y gives

$$x = (c_4 z_0)^{1/p}, \qquad y = (c_4 z_0/c_3)^{1/q} \tag{10}$$

Further substitution of Eq. (10) into the expression for r_0 from Eqs. (9) gives

$$r_0^2 = (c_4 z_0)^{2/p} + (c_4 z_0/c_3)^{2/q} \tag{11}$$

Eliminating the constants c_3 and c_4 from Eq. (11) by use of Eqs. (8) produces the equation for the thermal-stress-free interface between two orthotropic materials:

$$\left(\frac{x}{r_0}\right)^2 \left(\frac{z_0}{z}\right)^{2/p} + \left(\frac{y}{r_0}\right)^2 \left(\frac{z_0}{z}\right)^{2/q} = 1 \tag{12}$$

For transversely isotropic materials ($p = q$), Eq. (12) can be converted to cylindrical coordinates and reduced to the following form, which is identical to the equation developed by Blosser and McWithey[2]:

$$z = (z_0/r_0^p)r^p \tag{13}$$

For isotropic materials, Eqs. (12) and (13) reduce to the equation for a conical surface, which applies to the fastener described by Jackson et al.[3]

Equation (12) represents only one of a family of surfaces which provide a thermal-stress-free interface between two orthotropic materials. Equation (12) is limited to a surface containing the circular curve defined in Eqs. (9). Equations (9) could be replaced by any desired curve, which could then be combined with Eqs. (8) to define a thermal-stress-free interface. If Eqs. (8) and the specified curve cannot be readily combined algebraically, they may be easily solved numerically to map out points on the surface.

Discussions of Assumptions

In the previous analysis, the following assumptions were made: no frictional forces act along the interface between the materials; the principal material axes of the two materials are aligned and parallel to the coordinate system; motion resulting from thermal expansion of both materials is relative to the origin of the coordinate system; the CTE of each material is independent of temperature and location within the material; and the temperature is uniformly increased in both materials.

Frictional forces have been ignored to simplify the analysis. For an interface between two materials which is shaped according to Eq. (12), the thermal expansion will not produce any normal forces across that interface to induce friction. In the absence of adherence between the surfaces or applied mechanical forces normal to the interface, there should be no frictional forces.

The material axes of the two materials have been assumed to be aligned to simplify the expressions for motion due to thermal expansion. The CTE's are described by a symmetric second-order tensor,[6] which by definition has principal directions. Therefore, it should be possible to align the principal axes of any two homogeneous materials. In fact, because the CTE's are a second-order tensor for completely anisotropic, homogeneous materials, the present development is directly applicable to anisotropic homogeneous materials.

The motion due to thermal expansion in both materials is assumed to be relative to a common point. The fastener and washer must be arranged to make this assumption correct.

The CTE's for each material are assumed to be constant with temperature. In many real materials, however, the thermal expansion varies considerably with temperature. The interface between materials can be shaped to be in full contact, but thermal-stress-free, at the fabrication temperature and at a particular but arbitrary design temperature by using the average coefficients of thermal expansion between the two temperatures. The assembly will have to be analyzed for other temperatures to determine if the resulting separation along the interface and/or thermal stresses is acceptable.

Both materials were assumed to be at a common, uniform temperature. For the derivation of the initial shape of the interface at $T = T_0$, the implied assumption is that the temperature increases uniformly everywhere in both materials. The materials would most likely be shaped and assembled at a uniform temperature (probably room temperature). Uniformly increasing the temperature would, therefore, result in a uniform elevated temperature. In many applications, temperature gradients may develop, and the joint must be analyzed to determine if the resulting thermal stresses and/or separations along the interface are acceptable. If these conditions are unacceptable, it may be possible to shape an interface to be thermal-stress-free for a specific temperature distribution by modifying expressions [Eqs. (3)] to account for the spatial variation of temperature change required to go from uniform room temperature to the design temperature distribution. The resulting shape would be thermal-stress-free only for that specific temperature distribution, however.

Characteristic Shapes

The shape of the thermal-stress-free interface defined by Eq. (12) is governed by the parameters p and q, which are functions of the CTE's of the two materials. The curves produced from the intersection of the surface described by Eq. (12) and the x-y and y-z planes are identical to the characteristic shapes given by Blosser and McWithey[2] and shown in Fig. 5. Thus, the values of p and q determine the characteristic shape of the surface and the suitability of the surface for use as a fastener. The first three characteristic shapes shown on Fig. 5 are those which can be most readily applied to fasteners.

A typical thermal-stress-free joint is shown in Fig. 3. The joint consists of a fastener which has its bearing surface shaped according to Eq. (12), a matching hole in the sheets of joined material, and a spacer with the same thermal expansion properties as the joined material. The shaped head of the fastener is mated to a cylindrical shank. A clearance is necessary around the cylindrical shank within the spacer. The horizontal interface between the spacer and fastener material assures that the thermal expansion will occur relative to the origin of the coordinate system, as assumed in the analysis.

Figures 6a and 6b show various thermal-stress-free fastener shapes for different values of p and q greater than zero. The thermal-stress-free shapes are joined to a cylindrical shank that reaches to the origin of the coordinate system. Figure 6a shows a typical conical fastener that would be thermal-stress-free for isotropic materials ($p = q = 1$), and axisymmetric fasteners that would be thermal-stress-free for transversely isotropic materials ($p = q$). The shapes in Fig. 6a are identical to those described by Blosser and McWithey.[2] Figure 6b shows thermal-stress-free shapes for various combinations of p and q. These shapes are essentially combinations of the characteristic shapes in Fig. 6a. However, the shapes are not axisymmetric and would, therefore, be more difficult to fabricate. Not all combinations of materials have thermal-stress-free interfaces that are feasible for use as fasteners. Figure 6c shows examples of several shapes that are not feasible for fasteners (either $p<0$ or $q<0$). The upper fastener has no means to provide clamping forces, and the other two fasteners cannot be inserted into similarly shaped holes. The shapes shown in Fig. 6 were shaped to mate with a cylindrical shank. It is possible, as stated earlier, to mate the thermal-stress-free portion of the fastener with almost any shape of shank. For a different shape of shank, the fastener shape could be much different; however, the characteristic shape in the x-z and y-z planes would be the same. Also, the radius and length of the cylindrical shank of the fastener provide a designer with some control over the dimensions of the thermal-stress-free fastener.[2]

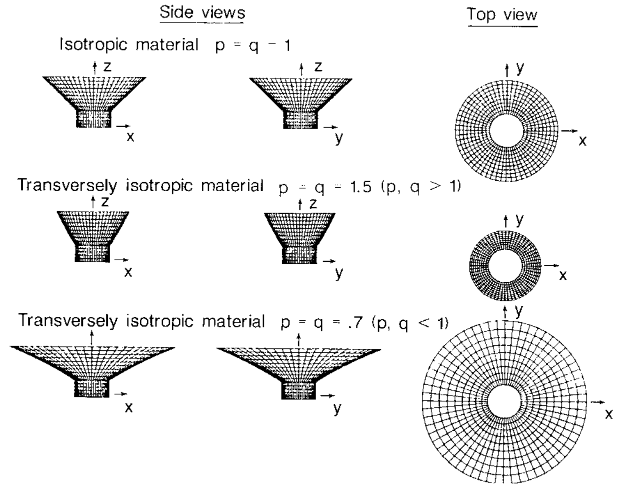


Fig. 6a Typical thermal-stress-free fastener shapes for isotropic and transversely isotropic materials.

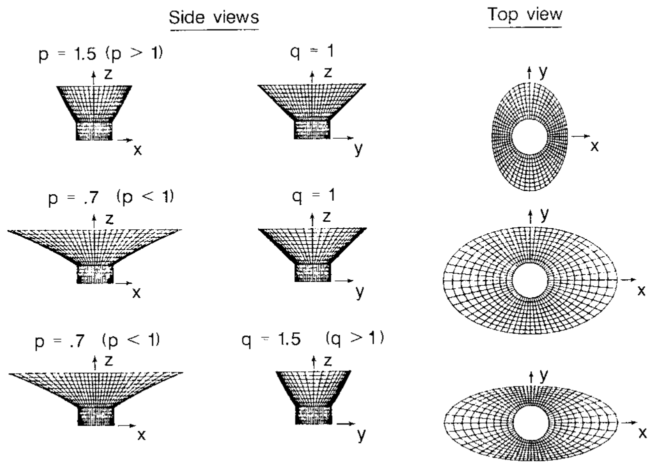


Fig. 6b Typical thermal-stress-free fastener shapes for orthotropic materials.

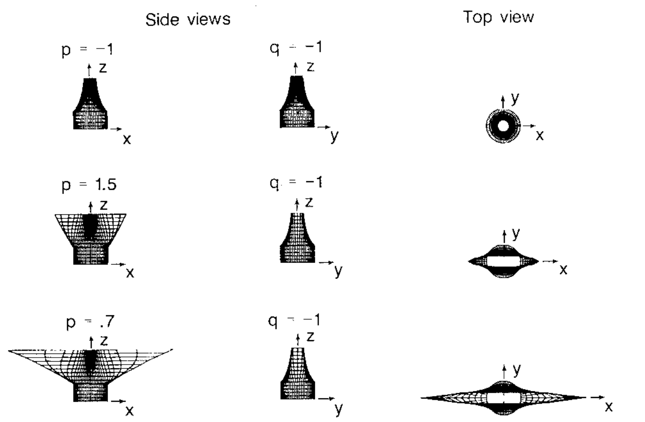


Fig. 6c Thermal-stress-free fastener shapes impractical for some material combinations.

Verification of Thermal-Stress-Free Shapes

It remains to be demonstrated that two materials with an interface shaped according to Eq. (12) will remain in contact without thermal stresses at any temperature. This can be shown as follows.

Consider the thermal expansion of each material separately. If the CTE's are assumed to be independent of temperature the exact expression[2] for thermal expansion can be found by integrating the equation

$$dL = L\alpha \, dT \tag{14}$$

to produce

$$L = L_0 e^{\alpha(T - T_0)} \tag{15}$$

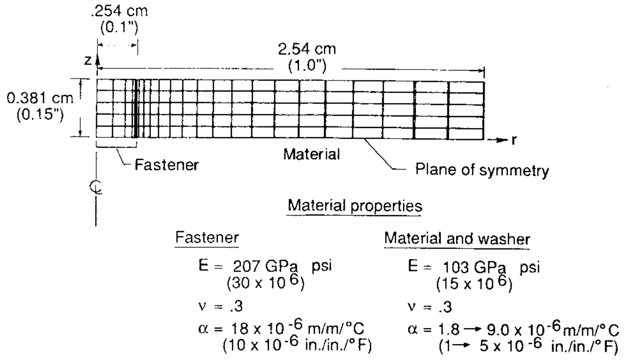


Fig. 7 Finite-element model of cylindrical fastener and surrounding material.

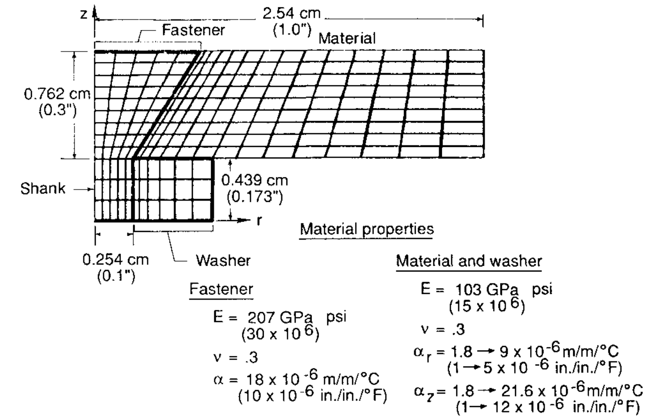


Fig. 8 Finite-element model of conical fastener and surrounding material.

The equations for the location of points in material 1 after expansion are, therefore,

$$x_1 = xe^{\alpha_{x1}(T - T_0)}, \quad y_1 = ye^{\alpha_{y1}(T - T_0)}, \quad z_1 = ze^{\alpha_{z1}(T - T_0)} \tag{16}$$

Similarly, for material 2,

$$x_2 = xe^{\alpha_{x2}(T - T_0)}, \quad y_2 = ye^{\alpha_{y2}(T - T_0)}, \quad z_2 = ze^{\alpha_{z2}(T - T_0)} \tag{17}$$

The location of the boundary of material 1 at an arbitrary temperature T can be found by substituting Eqs. (16) into Eq. (12), replacing the x, y, and z in Eq. (12) by the expressions for x_1, y_1, and z_1. Similarly, the location of the boundary of material 2 at temperature T can be found by substituting Eqs. (17) into Eq. (12). The shape of the two boundaries at temperature T is found to be the same for both materials and is given by

$$\left(\frac{x}{r_0}\right)^2\left(\frac{z_0}{z}\right)^{2/p} e^{m(T - T_0)} + \left(\frac{y}{r_0}\right)^2\left(\frac{z_0}{z}\right)^{2/q} e^{n(T - T_0)} = 1 \tag{18}$$

where

$$m = \frac{\alpha_{x_1}\alpha_{z_2} - \alpha_{x_2}\alpha_{z_1}}{\alpha_{z_2} - \alpha_{z_1}}$$

$$n = \frac{\alpha_{y_1}\alpha_{z_2} - \alpha_{y_2}\alpha_{z_1}}{\alpha_{z_2} - \alpha_{z_1}}$$

The boundaries are coincident at any temperature T and, therefore, the two materials remain in contact, and yet do not develop thermal stresses.

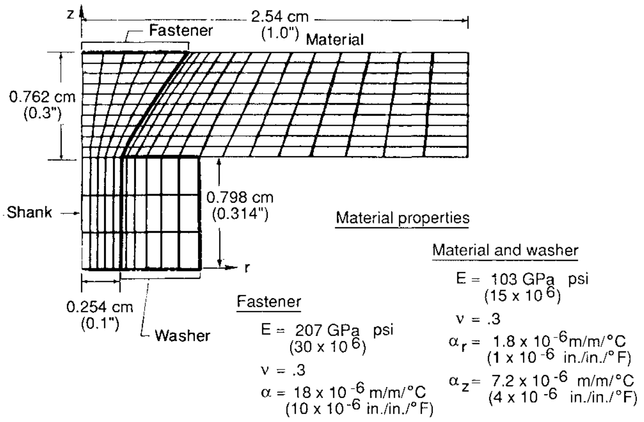


Fig. 9 Finite-element model of thermal-stress-free fastener for transversely isotropic material.

Finite-Element Analyses

The finite-element method is commonly used to analyze the stresses in joints numerically. Therefore, finite-element solutions were obtained for comparison with the mathematically derived solutions presented in this study. Finite-element analysis was used for two purposes in this study: to verify that a typical joint with a fastener shaped using Eq. (12) maintains contact and yet is thermal-stress-free at elevated temperature, and to investigate the thermal stresses that result from using a conical fastener (thermal-stress-free only with isotropic materials) to join a material with transversely isotropic CTE's. An existing, general-purpose finite-element program, Engineering Analysis Language[7] (EAL), was used for the analyses.

Modeling

Three-dimensional finite-element models were used to represent several joint configurations with isotropic and transversely isotropic materials. For the axisymmetric fasteners, a 5-deg wedge-shaped model of the fastener and surrounding material was used. The three joints that were modeled are shown in Fig. 7–9. Figure 7 shows the finite-element model of a cylindrical fastener and surrounding material, Fig. 8 shows the model of a conical fastener with washer and surrounding material, and Fig. 9 shows an axisymmetric fastener shaped to be thermal-stress-free for transversely isotropic material.

Particular, but arbitrary, dimensions were chosen for each of the three joints modeled. These dimensions are shown on Figs. 7–9. The half-angle of the conical fastener was 60 deg, and the thickness of the washer shown in Fig. 8 was sized so that the vertex of the conical surface was located on the bottom plane of the model. The fastener in Fig. 9 was shaped according to Eq. 13, so that its maximum and shank radii would be the same as the conical fastener in Fig. 8. The thickness of the washer was sized so that the origin of the coordinate system was located on the lower surface of the model. The radii of the washers in Figs. 8 and 9 were sized to be slightly larger than the maximum radii of the respective fasteners.

Three-dimensional, linear 6-node and 8-node elements were used to model the fastener, and 8-node elements were used to model the surrounding material. Nodes on the sides and bottons of the models were constrained to remain in their respective planes but were free to move within those planes.

The most challenging portion of the finite-element analysis was modeling the contact between the fastener and surrounding material along the bearing surface. Adjacent nodes along this boundary were connected by zero-length elements that had high stiffness perpendicular to the bearing surface and no stiffness tangent to the surface. The elements allowed relative motion along the boundary between the adjacent nodes but

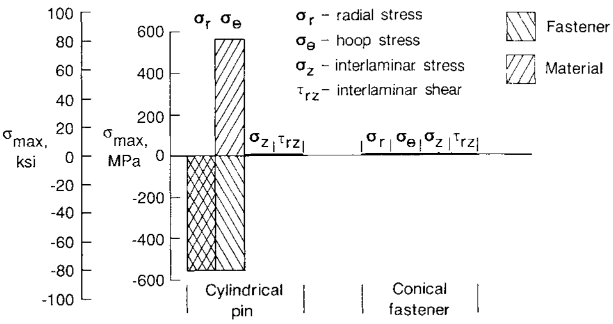


Fig. 10 Thermal stresses in a joint with isotropic material ($p = 1$).

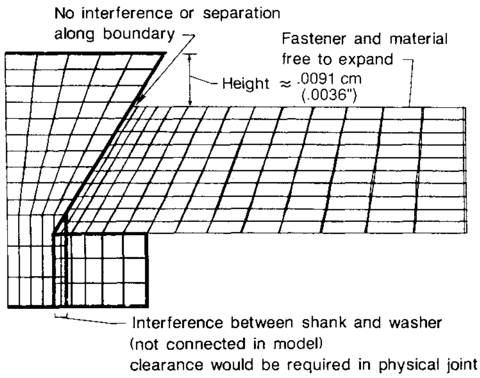


Fig. 11 Thermally expanded shape of finite-element model of conical fastener for isotropic material (thermal expansion exaggerated).

not perpendicular to it. The model was set up so that, if one of these elements was found to be in tension, the element stiffness could be reduced to an insignificant value and the two adjacent nodes allowed to separate. The contact between the washers and the material surrounding the fasteners in Figs. 8 and 9 was modeled in the same way. The clearance between the shank and the washer was simulated by providing no connection between the washer and shank in the radial direction. An arbitrary set of isotropic stiffness properties, indicated on Figs. 7-9, was used for all three models. The range of CTE, α, used for the fastener and surrounding material is also indicated.

A limitation of the finite-element program precluded modeling a thermal-stress-free joint for orthotropic materials shaped using Eq. (12). The program only allows definition of CTE's in the element reference frames and does not provide a means to specify CTE's in the global reference frame. It is impractical to construct a finite-element mesh for this problem so that all of the element reference frames are parallel to the global frame. In addition, the program does not provide a means of entering the full CTE matrix for each element, and so it was impossible to input the correct CTE matrix for each element to simulate principal expansions relative to the global reference frame.

Numerical Results

The three finite-element models, previously described, were used to calculate thermal stresses for several combinations of materials loaded by a uniform temperature increase of 556°C (1000°F). The geometry and loading are axisymmetric. Therefore, the r-θ shear and the θ-z shear stresses are zero.

The thermal stresses for a combination of isotropic materials in a cylindrical fastener joint and a conical fastener joint are shown in Fig. 10. The CTE of the fastener material is 18×10^{-6} m/m/°C (10×10^{-6} in./in./°F), and that of the material surrounding the fastener is 1.8×10^{-6} m/m/°C (1×10^{-6} in./in./°F) in all directions. As shown in the figure, the cylindrical fastener is in biaxial compression of just over -550 Mpa ($-80{,}000$ psi). The material around the cylindrical fastener has compressive bearing stress and a tensile hoop stress of approximately the same magnitude. Stresses in the z direction are negligible. The conical fastener model, loaded by the same 556°C (1000°F) temperature increase, showed negligible thermal stresses. The expanded shape of the conical fastener joint in isotropic material is illustrated in Fig. 11. The conical fastener expands more than the surrounding material, and yet the two materials on the conical interface do not separate or interfere. Thermal growth is accommodated by sliding along the conical interface, so that both materials can expand freely. Although the exaggerated thermal expansion appears large on the figure, the actual difference in vertical growth is only 0.0091 cm (0.0036 in.), which is small compared to fastener dimensions. Interference does occur between the fastener shank and the washer, as anticipated. However, they are not connected in the finite-element model to simulate the clearance that would be required in a physical joint to avoid this interference.

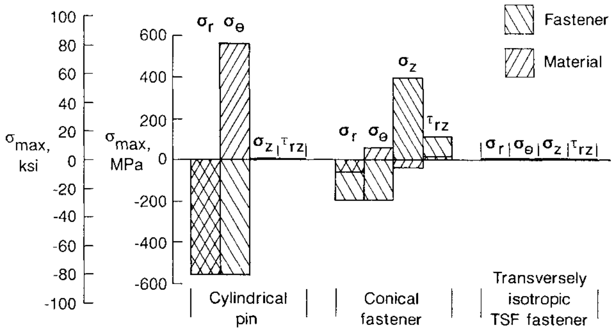


Fig. 12 Thermal stresses in a joint with transversely isotropic material ($p = 0.667$).

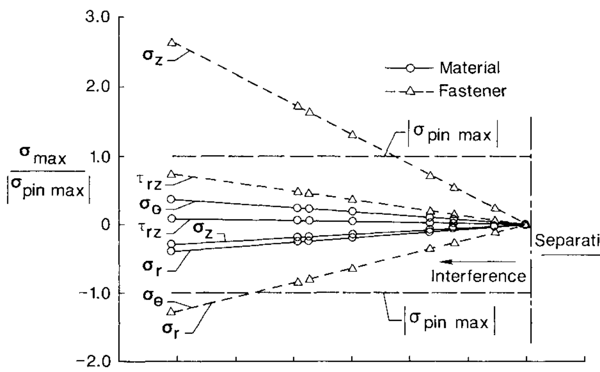


Fig. 13 Thermal stresses in a conical fastener joint with transversely isotropic material.

Thermal stresses in joints with transversely isotropic materials are shown in Fig. 12 for cylindrical pin, conical shape, and thermal-stress-free shaped fastener. The properties of the materials and the geometries of the cylindrical and conical fastener joints are the same as those for Fig. 10, except that the CTE through the thickness of the material surrounding the fastener is 7.2×10^{-6} m/m/°C (4×10^{-6} in./in./°F) ($p = 0.667$). The stresses in the cylindrical fastener joint are identical to those in Fig. 10. The conical fastener, however, now has nonzero stresses, because the materials are transversely isotropic. Note, however, that the thermal stresses in the conical fastener joint are lower than those in the cylin-

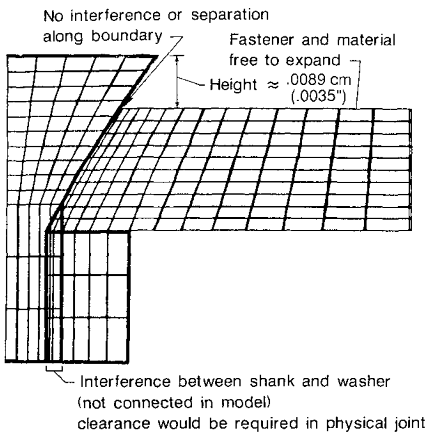


Fig. 14 Thermally expanded shape of finite-element model of TSF fastener for transversely isotropic material (thermal expansion exaggerated).

drical fastener for these particular dimensions and material properties. In addition, the fastener stresses are significantly higher than those in the surrounding material. The z and r-z shear stresses, which are negligible in the cylindrical fastener joint, are significant in the conical fastener joint. Figure 12 also shows that a fastener shaped according to Eq. (13) can eliminate the thermal stresses in a transversely isotropic joint.

Figure 13 shows the variation of thermal stresses in the conical fastener joint as a function of the parameter p, which depends on the CTE's of the two materials and determines the characteristic shapes for a thermal-stress-free shape (see Fig. 5). The stresses are normalized by the absolute value of the peak stress in the cylindrical fastener joint. When $p = 1$, the conical fastener is the correct thermal-stress-free shape, and the thermal stresses are zero. For $p > 1$, the fastener and material separate, and the fastener becomes loose. For $p < 1$, the fastener and material interfere with each other, and thermal stresses develop. These normalized stresses were found to be a linear function of p for the cases analyzed. The slope of these lines depends on the particular geometry and set of stiffness properties used. The horizontal dashed lines indicate the level of peak stress in the cylindrical fastener joint for comparison with the conical fastener stresses. Since the stresses are small when p is close to 1, it may be desirable to use conical fasteners to reduce thermal stresses to an acceptable level rather than using a more complex shaped fastener to eliminate thermal stresses.

Figure 14 shows the expanded shape resulting from thermal expansion of a fastener shaped to be thermal-stress-free in transversely isotropic material. As in Fig. 11, both materials are free to expand, and yet remain in contact on the shaped portion of the interface between them. Again, the shank and washer interfere, indicating the need for a clearance between them in a physical joint.

Concluding Remarks

High-temperature carbon and ceramic-based materials have significantly lower coefficients of linear thermal expansion (CTE's) than metals that are being considered for fasteners. A method was developed to define the shape of the joint interface between dissimilar orthotropic fastener and structural materials which will eliminate joint thermal stresses and maintain a snug fit over a wide range of uniform temperature distributions. A partial differential equation, which governs the shape of such an interface, was derived and solved to produce an algebraic equation for the shape of a thermal-stress-free fastener for orthotropic materials. This solution was also shown to apply to completely anisotropic materials. This equation was shown to reduce to a conical fastener for isotropic materials and to an axisymmetric fastener for transversely isotropic materials. The shape defined by the algebraic equation was mathematically shown to be thermal-stress-free, and yet remain snug at all temperatures for the assumptions made in the analysis. The following simplifying assumptions were made: no frictional forces act along the interface between the materials; the principal material axes of the two materials are aligned; the CTE of each material is independent of temperature and location within the material; and the temperature changes uniformly in both materials. The motion resulting from thermal expansion of both materials was assumed to be relative to the origin of the coordinate system.

The finite-element method is commonly used to numerically analyze the stresses in joints. Therefore, finite-element solutions were obtained for comparison with the mathematically derived solutions presented in this study. Finite-element analysis was used for two purposes in this study: to verify that a typical joint with a fastener, shaped to eliminate thermal stresses, maintains contact, and yet is thermal-stress-free at elevated temperature; and to investigate the thermal stresses that result from using a conical fastener to join a material with transversely isotropic CTE's. Finite-element calculations verified that a conical fastener can be thermal-stress-free for isotropic materials and that a properly shaped axisymmetric fastener can eliminate thermal stresses for transversely isotropic materials. A conical fastener joint in transversely isotropic materials develops through-the-thickness thermal stresses not found in cylindrical fastener joints. The thermal stresses for such a conical fastener joint were found to be significantly higher in the fastener than in the surrounding material for the geometry analyzed.

The results of this study indicate that cylindrical fasteners may cause high thermal stress, but fasteners can be shaped to eliminate thermal stresses while maintaining a snug fit for three-dimensional, homogeneous, orthotropic materials. Conical fasteners, which are thermal-stress-free for isotropic materials, can reduce thermal stresses for some transversely isotropic materials compared to a cylindrical fastener.

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